J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

TOPIC : CONTINUITY + REGRESSION   SYJC TEST - 02 - SET 1   DURATION - 1 $^1/_2$ HR   MARKS - 40

SOLUTION SET
SECTION - I

Q-1

01. $f(x) = \dfrac{x^2 - 3x + 2}{x - 1}$; $x \neq 1$

$= 3$; $x = 1$

Discuss continuity at x = 1

SOLUTION :

STEP 1

$\lim\limits_{x \to 1} f(x)$

$= \lim\limits_{x \to 1} \dfrac{x^2 - 3x + 2}{x - 1}$

$= \lim\limits_{x \to 1} \dfrac{\cancel{(x - 1)}(x - 2)}{\cancel{x - 1}}$ $x - 1 \neq 0$

$= \lim\limits_{x \to 1} x - 2$

$= 1 - 2$

$= -1$

STEP 2 :

$f(1) = 3$ ……… given

STEP 3 :

$f(1) \neq \lim\limits_{x \to 1} f(x)$

$\therefore$ f is discontinuous at x = 1

STEP 4 :
REMOVABLE DISCONTINUITY

f can be made continuous at x = 1 by redefining it as

$f(x) = \dfrac{x^2 - 3x + 2}{x - 1}$; $x \neq 1$

$= -1$; $x = 1$

02. $f(x) = \dfrac{\log(1 + 2x)}{x}$; $x \neq 0$

$= 2$; $x = 0$

Discuss continuity at x = 0

SOLUTION :

STEP 1

$\lim\limits_{x \to 0} f(x)$

$= \lim\limits_{x \to 0} \dfrac{\log(1 + 2x)}{x}$

$= \lim\limits_{x \to 0} \dfrac{2 \log(1 + 2x)}{2x}$

$= 2(1)$

$= 2$

STEP 2 :

$f(0) = 2$ ……… given

STEP 3 :

$f(0) = \lim\limits_{x \to 0} f(x)$

$\therefore$ f is continuous at x = 0

03. $f(x) = \dfrac{e^{2x} - 1}{5x}$; $x \neq 0$

$= 2$; $x = 0$

Discuss continuity at x = 0

SOLUTION :

STEP 1

$\lim\limits_{x \to 0} f(x)$

$= \lim\limits_{x \to 0} \dfrac{e^{2x} - 1}{5x}$

$= \lim\limits_{x \to 0} \dfrac{1}{5} \dfrac{e^{2x} - 1}{x}$

$= \lim\limits_{x \to 0} \dfrac{2}{5} \dfrac{e^{2x} - 1}{2x}$

$= \dfrac{2.\log e}{5}$

$= 2/5$

STEP 2 :

$f(0) = 2$ ……… given

STEP 3 :

$f(0) \neq \lim_{x\to 0} f(x)$

$\therefore f$ is discontinuous at $x = 0$

STEP 4 :

REMOVABLE DISCONTINUITY

f can be made continuous at $x = 0$ by redefining it as

$$f(x) = \frac{e^{2x} - 1}{5x} \quad ; \quad x \neq 0$$

$$= 2/5 \quad ; \quad x = 0$$

04. $f(x) = \dfrac{\sin^2 x}{1 - \cos^3 x} \quad ; \quad x \neq 0$

$= 3/2 \quad ; \quad x = 0$

Discuss continuity at $x = 0$

STEP 1

$\lim_{x\to 0} f(x)$

$= \lim_{x\to 0} \dfrac{\sin^2 x}{1 - \cos^3 x}$

$= \lim_{x\to 0} \dfrac{1 - \cos^2 x}{(1 - \cos x)(1 + \cos x + \cos^2 x)}$

$= \lim_{x\to 0} \dfrac{\cancel{(1 - \cos x)}(1 + \cos x)}{\cancel{(1 - \cos x)}(1 + \cos x + \cos^2 x)}$

as $x\to 0$, $1 - \cos x \neq 0$

$= \lim_{x\to 0} \dfrac{1 + \cos x}{1 + \cos x + \cos^2 x}$

$= \dfrac{1 + \cos 0}{1 + \cos 0 + \cos^2 0}$

$= \dfrac{1 + 1}{1 + 1 + 1}$

$= \dfrac{2}{3}$

STEP 2 :

$f(0) = 1$ ……… given

STEP 3 :

$f(0) \neq \lim_{x\to 0} f(x)$

$\therefore f$ is discontinuous at $x = 0$

STEP 4 :

REMOVABLE DISCONTINUITY

f can be made continuous at $x = 0$ by redefining it as

$$f(x) = \frac{\sin^2 x}{1 - \cos^3 x} \quad ; \quad x \neq 0$$

$$= 2/3 \quad ; \quad x = 0$$

Q-2

Q2. Attempt any TWO of the following

(3 marks each) (6 marks)

01. $f(x) = x^2 + 1 \quad ; \quad x < 0$

$= 5\sqrt{x^2 + 1} + k \quad ; \quad x \geq 0$

find k if the f is continuous at $x = 0$

SOLUTION :

STEP 1

$\lim_{x\to 0-} f(x)$

$= \lim_{x\to 0} x^2 + 1$

$= 0^2 + 1 = 1$

STEP 2

$\lim_{x\to 0+} f(x)$

$= \lim_{x\to 0} 5\sqrt{x^2 + 1} + k$

$= 5\sqrt{0^2 + 1} + k$

$= 5 + k$

STEP 3

$$f(0) = 5\sqrt{0^2 + 1} + k$$

$$= 5 + k$$

STEP 4

Since f is continuous at x = 0

$$\lim_{x\to 0-} f(x) = \lim_{x\to 0+} f(x) = f(0)$$

$$1 = 5 + k = 5 + k$$

$$5 + k = 1$$

$$k = -4$$

02. $f(x) = \dfrac{6^x + 3^x - 2^x - 1}{x}$; $x \neq 0$

find f(0) if f is continuous at x = 0

SOLUTION :

STEP 1

$$\lim_{x\to 0} f(x)$$

$$= \lim_{x\to 0} \frac{6^x + 3^x - 2^x - 1}{x}$$

$$= \lim_{x\to 0} \frac{6^x - 1 + 3^x - 2^x}{x}$$

$$= \lim_{x\to 0} \frac{6^x - 1 + 3^x - 1 - 2^x + 1}{x}$$

$$= \lim_{x\to 0} \frac{6^x - 1 + 3^x - 1 - (2^x - 1)}{x}$$

$$= \lim_{x\to 0} \frac{6^x - 1}{x} + \frac{3^x - 1}{x} - \frac{2^x - 1}{x}$$

$$= \log 6 + \log 3 - \log 2$$

$$= \log\left(\frac{6.3}{2}\right)$$

$$= \log 9$$

STEP 2 :

Since f is continuous at x = 0

$$f(0) = \lim_{x\to 0} f(x)$$

$$= \log 9$$

03. $f(x) = \dfrac{\tan x - \sin x}{x^3}$; $x \neq 0$

find f(0) if f is continuous at x = 0

STEP 1

$$\lim_{x\to 0} f(x)$$

$$= \lim_{x\to 0} \frac{\tan x - \sin x}{x^3}$$

$$= \lim_{x\to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3}$$

$$= \lim_{x\to 0} \frac{\sin x - \sin x . \cos x}{\cos x . x^3}$$

$$= \lim_{x\to 0} \frac{\sin x (1 - \cos x)}{\cos x . x^3}$$

$$= \lim_{x\to 0} \frac{\tan x . 2\sin^2 \frac{x}{2}}{x^3}$$

$$= \lim_{x\to 0} \frac{\tan x}{x} . \frac{2\sin^2 \frac{x}{2}}{x^2}$$

$$= \lim_{x\to 0} \frac{\tan x}{x} . 2\left(\frac{\sin \frac{x}{2}}{x}\right)^2$$

$$= \lim_{x\to 0} \frac{\sin x}{x} . 2\left(\frac{\frac{1}{2}\sin \frac{x}{2}}{\frac{x}{2}}\right)^2$$

$$= 1 . 2\left(\frac{1}{2} . 1\right)^2$$

$$= \frac{1}{2}$$

STEP 2 :

Since f is continuous at x = 0

$$f(0) = \lim_{x\to 0} f(x)$$

$$= 1/2$$

Q - 3

Q3. Attempt any TWO of the following

(4 marks each) (8 marks)

01. if the f given below is continuous at x = 2 and x = 4 then find a & b

$$f(x) = x^2 + ax + b \quad ; \quad x < 2$$
$$= 3x + 2 \quad ; \quad 2 \le x \le 4$$
$$= 2ax + 5b \quad ; \quad 4 < x$$

SOLUTION :

PART – 1

STEP 1

$$\lim_{x\to 2-} f(x)$$

$$= \lim_{x\to 2} x^2 + ax + b$$

$$= 2^2 + a(2) + b$$

$$= 4 + 2a + b$$

STEP 2

$$\lim_{x\to 2+} f(x)$$

$$= \lim_{x\to 2} 3x + 2$$

$$= 3(2) + 2 = 8$$

STEP 3

$$f(2) = 3(2) + 2 = 8$$

STEP 4

Since the f is continuous at x = 2

$$\lim_{x\to 2-} f(x) = \lim_{x\to 2+} f(x) = f(2)$$

$$4 + 2a + b = 8 = 8$$

$$2a + b = 4 \quad \ldots\ldots\ldots\ldots\ldots. \quad (1)$$

PART – 2

STEP 1

$$\lim_{x\to 4-} f(x)$$

$$= \lim_{x\to 4} 3x + 2$$

$$= 3(4) + 2 = 14$$

STEP 2

$$\lim_{x\to 4+} f(x)$$

$$= \lim_{x\to 4} 2ax + 5b$$

$$= 2a(4) + 5b$$

$$= 8a + 5b$$

STEP 3

$$f(4) = 3(4) + 2 = 14$$

STEP 4

Since the f is continuous at x = 4

$$\lim_{x\to 4-} f(x) = \lim_{x\to 4+} f(x) = f(4)$$

$$14 = 8a + 5b = 14$$

$$8a + 5b = 14 \quad \ldots\ldots\ldots\ldots\ldots. \quad (2)$$

Solving (1) and (2) : a = 3 , b = –2

02. find f(0) if f is continuous at x = 0 where

$$f(x) = \frac{5^x + 5^{-x} - 2}{x^2} \quad ; \quad x \neq 0$$

SOLUTION :

STEP 1

$$\lim_{x\to 0} f(x)$$

$$= \lim_{x\to 0} \frac{5^x + 5^{-x} - 2}{x^2}$$

$$= \lim_{x\to 0} \frac{5^x + \frac{1}{5^x} - 2}{x^2}$$

$$= \lim_{x\to 0} \frac{(5^x)^2 + 1 - 2.5^x}{x^2 . 5^x}$$

$$= \lim_{x\to 0} \frac{(5^x - 1)^2}{x^2} \quad \frac{1}{5^x}$$

$$= \lim_{x\to 0} \left(\frac{5^x - 1}{x}\right)^2 \quad \frac{1}{5^x}$$

$$= (\log 5)^2 \ . \frac{1}{5^0}.$$

$$= (\log 5)^2$$

STEP 2

Since f is continuous at x = 0

$$f(0) = \lim_{x\to 0} f(x)$$

$$f(0) = (\log 5)^2.$$

03. $f(x) = \dfrac{1 - \sin x}{(\pi - 2x)^2}$; $x \neq \pi/2$

find $f(\pi/2)$ if f is continuous at $x = \pi/2$

Step 1

$$\lim_{x\to \pi/2} f(x)$$

put $x = \dfrac{\pi}{2} + h$

$$= \lim_{h\to 0} \frac{1 - \sin\left(\frac{\pi}{2} + h\right)}{\left(\pi - 2\left(\frac{\pi}{2} + h\right)\right)^2}$$

$$= \lim_{h\to 0} \frac{1 - \cos h}{(\pi - \pi - 2h)^2}$$

$$= \lim_{h\to 0} \frac{2 \sin^2\left(\frac{h}{2}\right)}{4h^2}$$

$$= \lim_{h\to 0} \frac{2}{4} \frac{\sin^2\left(\frac{h}{2}\right)}{h^2}$$

$$= \lim_{h\to 0} \frac{2}{4} \left(\frac{\sin \frac{h}{2}}{h}\right)^2$$

$$= \lim_{h\to 0} \frac{2}{4} \left(\frac{\frac{1}{2}\sin \frac{h}{2}}{\frac{h}{2}}\right)^2$$

$$= \frac{2}{4} \left(\frac{1}{2} . 1\right)^2$$

$$= \frac{1}{8}$$

Step 2 :

Since f is continuous at $x = \pi/2$

$$f(\pi/2) = \lim_{x\to \pi/2} f(x)$$

$$= \frac{1}{8}$$

Q - 3

Q-4

SECTION - II

Q4. Attempt any THREE of the following

(2 marks each) (6 marks)

01. Values of two regression coefficients between the variables X and Y are byx = – 0.4 and bxy = – 2.025 respectively . Obtain the value of correlation coefficient

SOLUTION

$r^2 = \text{byx} \times \text{bxy}$

$r^2 = -0.4 \times -2.025$

$r^2 = \frac{4}{10} \times \frac{2025}{1000}$

$r^2 = \frac{8100}{10000}$

$r^2 = \frac{81}{100}$

$r = \pm \frac{9}{10}$

$r = -\frac{9}{10}$ (byx & bxy are –ve)

02. for a bivariate data ;

$\bar{x} = 53$, $\bar{y} = 28$, byx = – 1.5 , bxy = –0.2 . Estimate of y for x = 50

SOLUTION

$y - \bar{y} = \text{byx}(x - \bar{x})$

$y - 28 = -1.5(x - 53)$

$y - 28 = -1.5(50 - 53)$

$y - 28 = -1.5(-3)$

$y - 28 = 4.5$

$y = 32.5$ for $x = 50$

03. from the data of 20 pairs of observations on X and Y following results are obtained

$\bar{x} = 199$; $\bar{y} = 94$; $\Sigma(x - \bar{x})^2 = 1298$

$\Sigma(y - \bar{y})^2 = 600$; $\Sigma(x - \bar{x})(y - \bar{y}) = -262$.

Obtain regression coefficient bxy

SOLUTION

$$\text{bxy} = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\Sigma(y - y)^2}$$

$$= \frac{-262}{600}$$

$$= 0.4366$$

LOG CALC

2.4183
– 2.7782

AL $\bar{1}.6401$

0.4366

04. The equation of the two regression lines are

3x + 2y – 26 = 0 and 6x + y – 31 = 0 .

Find mean's

SOLUTION

$$\begin{aligned} 2 \times \quad 3x + 2y &= 26 \\ 6x + y &= 31 \end{aligned}$$

$$\begin{aligned} 6x + 4y &= 52 \\ -\quad 6x + y &= 31 \\ \hline 3y &= 21 \end{aligned}$$

$\bar{y} = 7$

subs in (1) $\bar{x} = 4$

Q5. Attempt any TWO of the following

(3 marks each)

Q - 5

01. you are given below the following information about advertising and sales

	Adv. Exp. (x)	sales (y)	in lacs
Mean	10	90	
S.D.	3	12	r = 0.8 .

What is the likely sales when adv. budget is ☐ 15 lacs

SOLUTION **y on x**

$$byx = r \cdot \frac{\sigma y}{\sigma x}$$

$$= 0.8 \times \frac{12}{3}$$

$$= 0.8 \times 4$$

$$= 3.2$$

$y - \overline{y} = byx(x - \overline{x})$

$y - 90 = 3.2(x - 10)$

$y - 90 = 3.2x - 32$

$y = 3.2x - 32 + 90$

$y = 3.2x + 58$

Put $x = 15$

$y = 3.2(15) + 58$

$y = 48 + 58$

$y = 106$ Estimated Sales = ☐ 106 lacs

03. Find the equation of line of regression of Y on X for the following data

$n = 8$; $\Sigma(x - \overline{x})(y - \overline{y}) = 120$; $\overline{x} = 20$ $\overline{y} = 36$; $\sigma x = 2$; $\sigma y = 3$

SOLUTION

$$\mathbf{byx} = \frac{cov(x,y)}{\sigma x^2}$$

$$= \frac{\frac{\Sigma((x - \overline{x})(y - \overline{y})}{n}}{\sigma x^2}$$

$$= \frac{\frac{120}{8}}{4} = \frac{15}{4} = 3.75$$

Y on X

$y - \overline{y} = byx\,(x - \overline{x})$

$y - 36 = 3.75(x - 20)$

$y - 36 = 3.75x - 75$

$y = 3.75x - 75 + 36$

$y = 3.75x - 39$

02. For group of 100 students the equation of regression line of marks in Economics (Y) on marks in Book keeping (X) is given by 4x − 3y − 120 = 0 . Ratio of variance of x to variance of y is 9 : 25 . Find measure of acute angle between the regression lines

SOLUTION :

GIVEN : Y ON X : $4x - 3y - 120 = 0$

$$\frac{\sigma x^2}{\sigma y^2} = \frac{9}{25}$$

STEP 1

Y ON X : $4x - 3y - 120 = 0$

$$3y = 4x - 120$$

$$y = \frac{4x}{3} - \frac{120}{3}$$

$$byx = \frac{4}{3}$$

STEP 2

$$byx = r \cdot \frac{\sigma y}{\sigma x}$$

$$\frac{4}{3} = r \times \frac{5}{3}$$

$$r = \frac{4}{5}$$

STEP 3

$$bxy = r \cdot \frac{\sigma x}{\sigma y}$$

$$= \frac{4}{5} \times \frac{3}{5}$$

$$= \frac{12}{25}$$

STEP 4

$$m_1 = byx = \frac{4}{3}$$

$$m_2 = \frac{1}{bxy} = \frac{25}{12}$$

STEP 5

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 . m_2}\right|$$

$$\tan\theta = \left|\frac{\frac{4}{3} - \frac{25}{12}}{1 + \frac{4}{3}\frac{25}{12}}\right|$$

$$\tan\theta = \left|\frac{48 - 75}{36 + 100}\right|$$

$$\tan\theta = \left|\frac{-27}{136}\right|$$

$$\tan\theta = \frac{27}{136}$$

$$\theta = \tan^{-1}\frac{27}{136}$$

Q6. Attempt any TWO of the following

(4 marks each)

Q-6

01. x : index of production
y : no. of unemployed (in lacs)

find regression line y on x

x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
100	15	– 4	0	16		0
102	12	– 2	– 3	4		6
104	13	0	– 2	0		– 0
107	11	3	– 4	9		–12
105	12	1	– 3	1		– 3
112	12	8	– 3	64		–24
103	19	– 1	4	1		– 4
99	26	– 5	11	25		–55
832	120	0	0	120		– 98 + 6 =–92
Σx $\bar{x}$ =104	Σy $\bar{y}$ = 15			$\Sigma(x - \bar{x})^2$	$\Sigma(y - \bar{y})^2$	$\Sigma(x - \bar{x})(y - \bar{y})$

$$byx = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\Sigma(x - x)^2}$$

$$= \frac{-92}{120}$$

$$= -0.77$$

$$y - \bar{y} = byx\,(x - \bar{x})$$

$$y - 15 = -0.77(x - 104)$$

$$y - 15 = -0.77x + 80.08$$

$$y = -0.77x + 80.08 + 15$$

$$y = -0.77x + 95.08$$

02.

The records of ten days is given

$\Sigma x = 580$; $\Sigma y = 370$; $\Sigma x^2 = 41658$; $\Sigma y^2 = 17206$; $\Sigma xy = 11494$

Obtain regression line Y on X

SOLUTION

$$byx = \frac{n\Sigma xy - \Sigma x.\Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{10(11494) - (580)(370)}{10(41658) - (580)^2}$$

$$= \frac{114940 - 214600}{416580 - 336400}$$

$$= \frac{-99660}{80180}$$

$$= -\frac{9966}{8018}$$

$$= -1.244$$

LOG CALC

3.9986
– 3.9040

AL 0.0946

1.244

$$y - \bar{y} = byx(x - \bar{x})$$

$$y - 37 = -1.244(x - 58)$$

$$y - 37 = -1.244x + 72.152$$

$$y = -1.244x + 72.152 + 37$$

$$y = -1.244x + 109.152$$

03. Regression of two series are **Q-6**

$$2x - y - 15 = 0 \quad \& \quad 3x - 4y + 25 = 0$$

Find the coefficient of correlation

STEP 1

ASSUME

XON Y : $2x - y - 15 = 0$

$$2x = y + 15$$

$$x = \frac{1}{2}y + \frac{15}{2}$$

$$bxy = \frac{1}{2}$$

Y ON X : $3x - 4y + 25 = 0$

$$4y = 3x + 25$$

$$y = \frac{3}{4}x + \frac{25}{4}$$

$$byx = \frac{3}{4}$$

STEP 2

$$r^2 = bxy \,.\, byx$$

$$= \frac{1}{2} \times \frac{3}{4}$$

$$= \frac{3}{8}$$

Since $0 \leq r^2 \leq 1$

Our assumptions are correct

$$r = \pm\sqrt{\frac{3}{8}}$$

$$r = +\sqrt{\frac{3}{8}} \quad \text{(byx \& bxy are + ve)}$$

$$\log r = \frac{1}{2}\left(\log 3 - \log 8\right)$$

$$\log r = \frac{1}{2}\left(0.4771 - 0.9031\right)$$

$$\log r = \frac{0.4771}{2} - \frac{0.9031}{2}$$

$$\log r = 0.2386 - 0.4516$$

$$\log r = \bar{1}\,.\,7870$$

$$r = AL(\bar{1}\,.\,7870\,)$$

$$r = 0.6124$$